

Mathematics: analysis and approaches
Standard Level
Paper 1

Name

Date: _____

1 hour 30 minutes

Instructions to candidates

- Write your name in the box above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all of Section A in the spaces provided.
- Section B: answer all of Section B on the answer sheets provided. Write your name on each answer sheet and attach them to this examination paper.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

exam: 10 pages



Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

Let $f(x) = \cos 4x$ and $g(x) = e^{3x-1}$

(a) Find $f'(x)$. [2]

(b) Find $g'(x)$. [2]

(c) Let $h(x) = g(x) \times f(x)$. Find $h'(x)$. [2]

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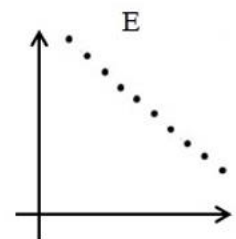
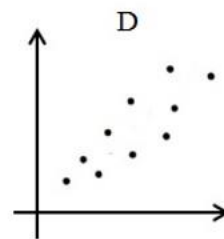
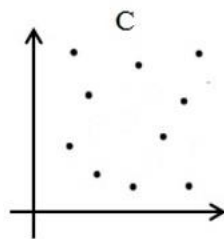
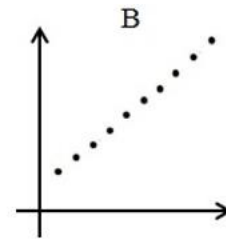
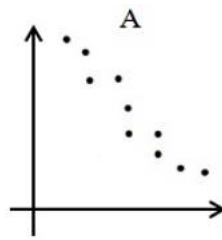
2. [Maximum mark: 6]

There are seven different plants being studied in a biology class. For each plant, x is the diameter of the stem in centimetres and y is the average leaf length in centimetres. Let r be the Pearson’s product-moment correlation coefficient.

(a) Write down the possible minimum and maximum values of r . [2]

(b) Copy and complete the following table by noting which scatter diagram A, B, C, D or E corresponds to each value of r . [4]

correlation coefficient r	scatter diagram
-1	
-0.8	
0	
0.5	



3. [Maximum mark: 5]

Let A and B be events such that $P(A) = 0.3$, $P(B) = 0.6$ and $P(A \cup B) = 0.7$. Find $P(A | B)$.

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4. [Maximum mark: 5]

Let n and $n+1$ be any two consecutive integers where $n \in \mathbb{Z}$. Hence, prove that the sum of the squares of any two consecutive integers is odd.

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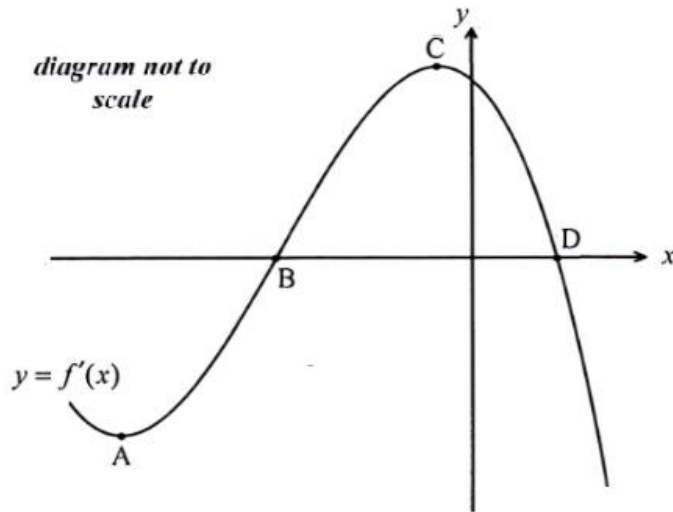
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5. [Maximum mark: 7]

The diagram shows part of the graph of $y = f'(x)$, the **derivative** of function f . The x -intercepts are at points B and D and there is a minimum at point A and a maximum at point C.



- (a) (i) Write down the value of $f'(x)$ at B. [3]
- (ii) Hence, verify that the x -coordinate of B is also the x -coordinate of a minimum on the graph of f . [3]
- (b) Which of the points A, C or D corresponds to a maximum on the graph of f ? [1]
- (c) Verify that C corresponds to a point of inflexion on the graph of f . [3]

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Question 5 continued on the next page

Question 5 continued

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6. [Maximum mark: 6]

A geometric series has a common ratio of 2^x .

- (a) Find the values of x for which the sum to infinity of the series exists. [2]
- (b) If the first term of the series is 14 and the sum to infinity is 16, find the value of x . [4]

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Do **not** write solutions on this page.

Section B

Answer **all** the questions on the answer sheets provided. Please start each question on a new page.

7. [Maximum mark: 13]

All of the students in a class of 35 must study at least one science – either Biology or Chemistry. Some of the students study both. 25 students study Biology and 15 students study Chemistry.

- (a) (i) Find the number of students who study both Biology and Chemistry
- (ii) Write down the number of students who study only Biology. [3]
- (b) One student is selected at random from the class.
- (i) Find the probability that the student studies only one science.
- (ii) Given that the student selected studies only one science, find the probability that The student studies Biology. [5]

Let B be the event that a student studies Biology and C be the event that a student studies Chemistry.

- (c) Show that B and C are **not** mutually exclusive. [2]
- (d) Show that B and C are **not** independent events. [3]

8. [Maximum mark: 16]

The function f is defined as $f(x) = \frac{x+1}{\ln(x+1)}$, $x > 0$.

- (a) (i) Show that $f'(x) = \frac{\ln(x+1)-1}{(\ln(x+1))^2}$.
- (ii) Find $f''(x)$, writing it as a single rational expression [6]
- (b) (i) Find the value of x satisfying the equation $f'(x) = 0$.
- (ii) Show that this value gives a minimum value for $f(x)$, and determine the minimum value of the function. [7]
- (c) Find the x -coordinate of the one point of inflexion on the graph of f . [3]

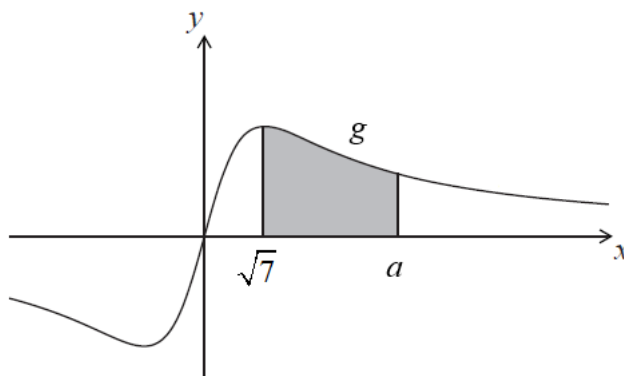
9. [Maximum mark: 16]

The function g is defined by $g(x) = \frac{3x}{x^2 + 7}$.

(a) Show that $g'(x) = \frac{21 - 3x^2}{(x^2 + 7)^2}$. [5]

(b) Find $\int \frac{3x}{x^2 + 7} dx$. [4]

The diagram below shows a portion of the graph of g .



(c) The shaded region is enclosed by the graph of g , the x -axis, and the lines $x = \sqrt{7}$ and $x = a$ such that $a > \sqrt{7}$. This region has an area of $\ln 8$. Find the value of a . [7]